Mr. Mohamed Amin Math Academy 3rd Sec **Revision Statics**

MATH ACADEMY

[1] A Body of weight 35 newton is placed on a rough horizontal plane , two horizontal forces act on the body of magnitudes 6 newton and 10 newton and including between them an angle of measure 60° , If the body is about to move , then the coefficient of static friction equals



- b) $\frac{1}{14}$
- c) $\frac{3}{7}$
- d) $\frac{1}{10}$



[2] If a body of weight 4 newton is placed on a rough horizontal plane, the coefficient of static friction between the plane and the body $=\frac{1}{4}$, and a horizontal force acts on the body trying to move it, then the static friction force \in

a)
$$\left[\frac{1}{4}, 4\right]$$

d)
$$\left[0,\frac{1}{4}\right]$$

[3] If The force $\vec{F} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ acts at point A(1, -1, 4) find the moment vector of the force \vec{F} about point (2, -3, 1), then calculate the length of the perpendicular drawn from the point B on the line of action of the force

$$m = (-1, 2, 3)$$
 , $\begin{vmatrix} i & j & k \\ -1 & 2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$

$$M_B = -11i - 5j - 7k$$
 , $L = \frac{\sqrt{121 + 25 + 49}}{\sqrt{4 + 9 + 1}} = 3.7$



[4] The forces $\overrightarrow{F_1} = 2\hat{\imath} - 3\hat{\jmath}$, $\overrightarrow{F_2} = 5\hat{\imath} - 2\hat{\jmath}$, $\overrightarrow{F_3} = -3\hat{\imath} + 2\hat{\jmath}$ act at the point A(-3,5), find the moment vector of the resultant of these the point B(1,7) and the distance between the point B and the line of action of the resultant

$$F = 4\vec{\imath} - 3\vec{\jmath}$$
 , $M = (-4, -2) \times (4, -3)$, $12 + 8 = 20$

$$L=\frac{20}{5}=4cm$$

MATH ACADEMY

a) 7

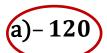


- c) 5
- d) 2

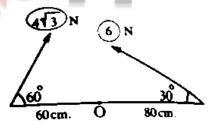
[6] In the Opposite Figure:

The sum of the moments of the forces about the point O equals

..... *N*. *cm*



- b) 120
- c) 240
- d) 360



MATH ACADEMY

[7] A body of weight400 gm . wt . is placed on rough plane inclined to the horizontal by an angle of measure 30° , the coefficient of static friction between it and the body is $\frac{\sqrt{3}}{4}$ A force of magnitude 50~gm . wt acts on it in the direction of the line of the the greatest slope of the plane upwards . If the body is in equilibrium , then determine the friction force and show whether the body is about to move or not ?

$$m=rac{\sqrt{3}}{4}$$
 , $w=400$,

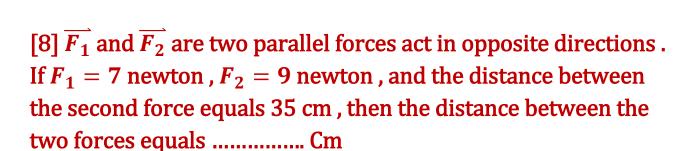
$$400 \sin 30^{\circ} > 50$$

$$F_s$$
 upward , $\frac{\sqrt{3}}{4} imes 400~cos~30^\circ = 150$

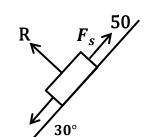
$$400 \sin 30^{\circ} = 50 + 150$$

$$200 = 200$$
 (About To Move)

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- a) 10
- b) 16
- c) 35
- d) 70



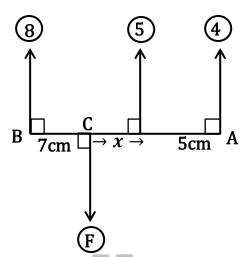
MATH ACADEMY

[9] In The Opposite Figure:

If \overline{AB} is a rod is equilibrium horizontally , then the distance

x = cm

- a)56
- b) 36
- c) 27
- d) 4



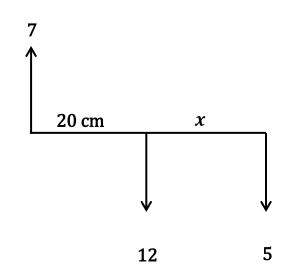
[10] Find the magnitude and the point of action of the resultant of two parallel forces of magnitude 7 newton, 12 newton act at A and B in the two opposite directions such that AB = 20cm

$$7(20+x)=12x$$

$$7 \times 20 = 12x - 7x$$

$$x=\frac{7\times20}{5}=28cm$$

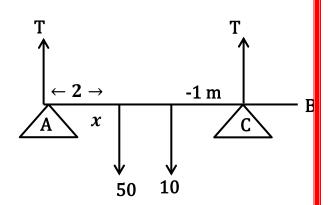
$$R = 12 - 7 = 5$$



MATH ACADEMY

[11] \overline{AB} is a uniform wooden board of mass 10kg. and length 4 meters rests horizontally on two supports one of them at A and the other at a point distant 1 meter from B show at which distance a 50~kg.wt. child can stand on the board in order to the reactions on the two supports get equal

$$2T = 60$$
 , $T = 30$
 $-50x - 10(2) + 30 \times 3 = 0$
 $-50x - 20 + 90 = 0$
 $-50x = -70$, $x = \frac{7}{5} = 1.4$ m



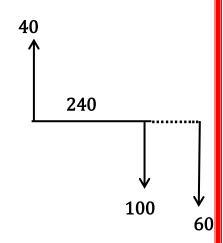
[12] Two parallel force of magnitudes 40, 100 newton act in opposite directions . If the distance between their lines of action equals 240cm, then find their resultant and its point of action

$$R = 100 - 4 = 60$$

$$40(240 + x) = 100x$$

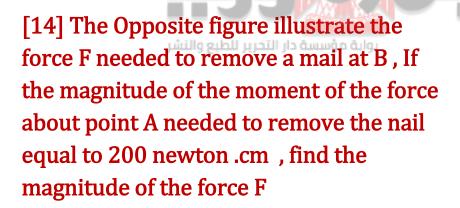
$$240 \times 40 = 60x$$

$$x = \frac{240 \times 40}{60} = 160 \text{ cm}$$



[13] Two forces $\overrightarrow{F_1}=2\hat{\imath}-\hat{\jmath}$, $\overrightarrow{F_2}=\hat{\jmath}-2\hat{\imath}$ act at two points A(1,1), B(0,-4) respectively. Find the moments of the system about any point on the plane

$$F_1 = (2, -1)$$
 , $F_2 = (-2, 1)$
 $(A(1, 1)$, $B(0, -4)$
 $M = (1, 1) \times (2, -1) + (0, -4) \times (-2, 1)$
 $-1 - 2 + (-8) = -11\vec{k}$

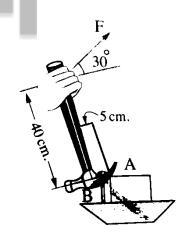


$$f \cos 3^{\circ} \times 40 + F \sin 30^{\circ} \times 5 = 200$$

$$\frac{\sqrt{3}}{2} F 40 + \frac{5}{2} F = 200$$

$$\left(20\sqrt{3} + \frac{5}{2}\right) F = 200$$

$$F = 5.38$$



MATH ACADEMY

[15] A body of weight 50 newton is placed on a rough inclined at an angle of measure θ to the horizontal . the least and greatest forces parallel to the line of the greatest slope and makes the body in equilibrium on the plane are 10 , 40 newton respectively . Find the coefficient of friction and the measure of the angle of inclination of the plane to the horizontal

$$10 = 50 \sin \theta - \mu (50 \cos \theta)$$

$$40 = 50 \sin \theta + \mu (50 \cos \theta)$$

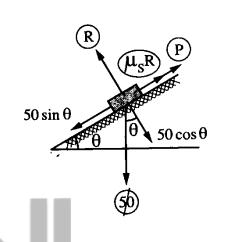
$$50 = 100 \sin \theta$$

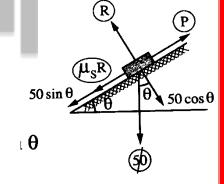
$$\sin \theta = \frac{1}{2}$$
 , $\theta = 30$

$$\mathbf{10} = \mathbf{25} - \mu(\mathbf{25}\sqrt{3})$$

$$15 = \mu(25\sqrt{3})$$
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$$\mu = \frac{\sqrt{3}}{5}$$





[16] If the force $\vec{F} = 2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ acts at point A whose vector is $\hat{\imath} - 3\hat{k}$, then the moment of \vec{F} about point b whose position vector is $\hat{\jmath} + 3\hat{k}$ is equal to

$$M = \begin{vmatrix} i & j & k \\ 1 & -1 & -6 \\ 2 & -1 & 5 \end{vmatrix} = -11 \,\vec{i} - 17 \vec{j} + \vec{k}$$

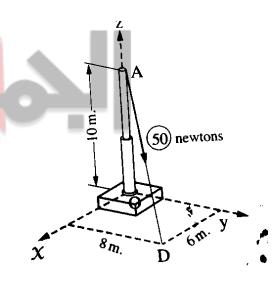
[17] The coefficient of the static friction is the ratio between

.....

(Limiting static friction force and normal reaction)

[18] In The Opposite Figure:

A force of magnitude 50 newton acts at point A, Find the moment of the force about 0



$$A(0,010)$$
 , $D(6,8,0)$

$$M = \begin{vmatrix} i & j & k \\ 0 & 0 & 10 \\ 15\sqrt{2} & 20\sqrt{2} & -25\sqrt{2} \end{vmatrix} -200\sqrt{2}\ \vec{\imath} + 150\sqrt{2}\vec{\jmath}$$

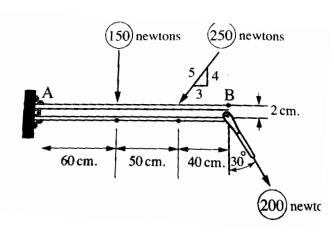
$$50\left(\frac{6}{10\sqrt{2}}\text{ ,}\frac{8}{10\sqrt{2}}\text{ ,}-\frac{10}{10\sqrt{2}}\right)$$

$$F = \left(\frac{30}{\sqrt{2}}, \frac{40}{\sqrt{2}}, -\frac{50}{\sqrt{2}}\right)$$
 , $\left(15\sqrt{2}, 20\sqrt{2}, -25\sqrt{2}\right)$

MATH ACADEMY

[19] In The Opposite Figure:

Three Coplanar force act on the rod \overline{AB} , find the algebraic measure of the sum of the moments of the forces about the two point A, B



$$M_A = -150 \times 60 - 250 \times \frac{4}{5} \times 110 - 200 \cos 30^{\circ} \times 150 + 200 \sin 30^{\circ} \times 2 = -56780.8 \ wt. \ cm$$

$$M_B = 20 \sin 30^{\circ} \times 2 + 250 \times \frac{4}{5} \times 40 + 150 \times 90$$

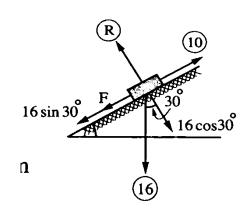
= 21700 kg.wt.cm

[20] A body of weight 16 kg. wt is placed on a plane inclined at 30° to the horizontal and the coefficient of friction between it and the body is equal to $\frac{1}{\sqrt{3}}$, A force in the line of the greatest slope of the plane acts on the body upwards and with magnitude 10 kg. wt. If the body is in equilibrium, identify the friction force and show whether the body is about to move or not

$$10=8+rac{F_s}{rac{1}{3} imes 16 imesrac{\sqrt{3}}{2}}$$
 , $F_s=2$

$$\mu N = \frac{1}{\sqrt{3}} \times 16 \times \cos 30^{\circ} = 8 \ kg \ .wt$$

(Not About To Move)



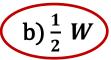
MATH ACADEMY

[21] In The Opposite Figure:

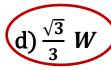
If the rod $\,$ is about to slide , then

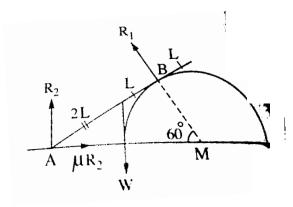
$$R_1 = \dots , R_2 = \dots$$

a)W



c)
$$\frac{\sqrt{3}}{2}$$
 W







[22] The coefficient of friction is based upon

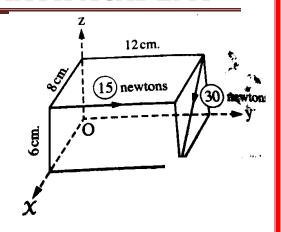
- a) the area of the constant surface between two bodies
- b) shape of the two bodies
- c) nature of the two bodies
 - d) all mentioned

MATH ACADEMY

[23] In The Opposite Figure:

Find the sum of moments of the forces about O

$$F_1 = 15 \left(0, \frac{12}{12}, 0 \right) = (0, 15, 0)$$



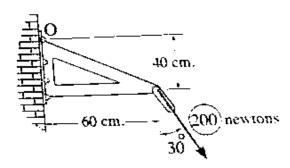
$$F_2 = 30 \left(rac{80}{10} , 0 , rac{6}{10}
ight) = (24, 0, -18)$$

$$M = \begin{vmatrix} i & j & k \\ 8 & 0 & 6 \\ 0 & 15 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 11 & 6 \\ 24 & 0 & -18 \end{vmatrix}$$



[24] In The Opposite Figure:

Find the moment of the force 200 newton about O



$$M_o = 200 \sin 30^{\circ} \times 40 - 200 \cos 30^{\circ} \times 60$$

= -6392.3 N.cm

MATH ACADEMY

[25] A body of weight $66\frac{2}{3}$ newton is placed on a rough horizontal plane . the coefficient of friction between them is equal to $\frac{3}{4}$, A force of magnitude 40 newton acts on the body and inclined at an acute angle of measure θ to the horizontal plane . what is the value of θ if the body is about to move ?

$$40 \cos \theta = \frac{3}{4} \left(66 \frac{2}{3} - 40 \sin \theta \right)$$

$$40\cos\theta = \frac{3}{4}R$$

$$R + 40 \sin = 66 \frac{2}{3}$$
 ,

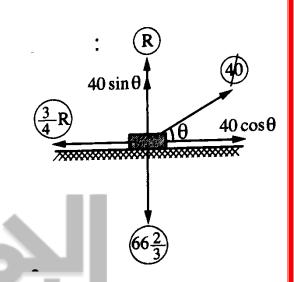
$$40 \sin \theta = 66 \frac{2}{3} - R$$

$$1600 \cos^2 \theta = \frac{9}{16} R^2 \ ,$$

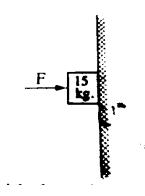
$$1600 \sin^2\theta = \left(6\frac{2}{3} - R\right)^2$$

$$1600 = \frac{9}{16} R^2 + \frac{40000}{9} - \frac{400}{3} R + R^2$$

$$R=rac{128}{3}$$
 , $cos \ heta=rac{4}{5}$, $heta=36^{\circ}52'$



MATH ACADEMY



ng.wt

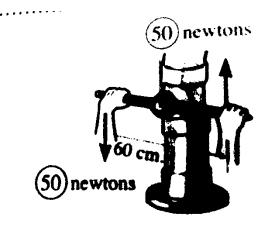
$$15 = \frac{1}{5} R$$
, $R = 15 \times 5 = 75$

[28] In The Opposite Figure:

The moment of the couple resulted from the two forces 50, 50 is equal to

.....

$$50 \times 60 = 3000 \, N \, . \, cm$$



MATH ACADEMY

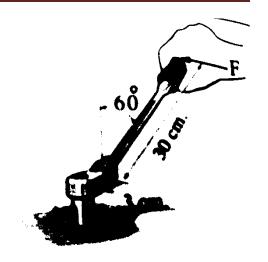
[29] In The Opposite Figure:

If the moment of the force F perpendicular to the rotation arm about point A is equal to 620 newton .cm , Find : F

$$F = \frac{620}{32.63} = 19 N$$

$$AB =$$

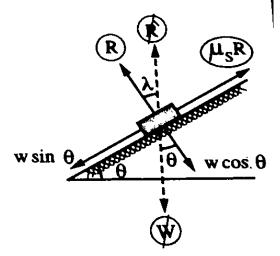
$$\sqrt{30^2 + 3^2 - 2 \times 30 \times 3 \cos 150} = 32.63$$





[30] Prove that if a body is placed on a rough inclined plane, and the body is about to slide, then the measure of the angle of friction is equal to the measure of the angle of inclination of the plane on the horizontal

$$w \sin \theta = \mu R$$
 $w \sin \theta = \mu w \cos \theta$
 $\tan \theta = \tan \lambda$
 $\theta = \lambda$



MATH ACADEMY

[31] If the force $\vec{F} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k}$ acts at the point A(-1, 2, 1)

Find: 1) The moment of the force \vec{F} about the origin point

2) The length of the perpendicular drawn from the origin point

On the line of action of \vec{F}

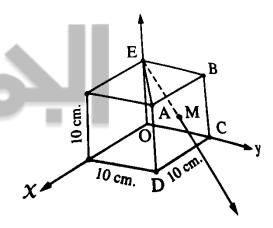
$$M_o = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 5 & -1 & 3 \end{vmatrix} = 7\vec{i} + 8\vec{j} - 9\vec{k}$$

$$L = \frac{\sqrt{7^2 + 8^2 + 9^2}}{\sqrt{5^2 + (-1)^2 + 32}} = 2.3 \ cm$$

[32] In The Opposite Figure:

A Force of magnitude $25\sqrt{6}$ newton acts at \overrightarrow{EM} where M is the

The geometric center of the square ABCD find the component of



The moment of the force about coordinates axes

$$E(0,0,10)$$
, $M(5,10,5)$

$$EM = (5, 10, -5), F = 25\sqrt{6}\left(\frac{5}{5\sqrt{6}}, \frac{10}{5\sqrt{6}}, -\frac{5}{5\sqrt{6}}\right)$$

$$(25, 50, -25)$$

$$M = \begin{vmatrix} i & j & k \\ 0 & 0 & 10 \\ 25 & 50 & -25 \end{vmatrix} - 500 \,\vec{i} + 250 \vec{j} + 0 \,\vec{k}$$

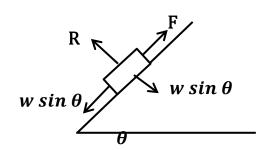
[33] A body of weight (W) is placed on a rough place inclined at an angle of measure (θ) to the horizon . If the measure of the angle of friction is (λ) , Find the magnitude and direction of the least force making the body about to move upwards

$$F = \mu w \cos \theta + w \sin \theta$$

$$\sin \lambda w \cos \theta - w \sin \theta \cos \lambda$$

$$\frac{\sin\lambda w\cos\theta}{\cos\lambda} + \frac{w\sin\theta\cos\lambda}{\cos\lambda}$$

$$F = \frac{w \, (\, \sin \theta + \lambda)}{\cos \lambda}$$
 , $\cos \lambda = 1$



[34] The Opposite Figure

illustrate a winch puller

 \overline{AB} acting on an inclined

fence $\overline{\textit{CD}}$, Find the

magnitude of the moment

of the tension force about point D

$$MD = -4 \times 6 + F_2(8)$$

$$F_2 = 3 N$$

MATH ACADEMY

[35] If θ is the measure of the angle between the limiting

Friction force and the resultant reaction, then the coefficient of

The static friction is equal to

- a) $tan \theta$
- b) $\sin \theta$
- c) $\cos \theta$
- (d) $\cot \theta$



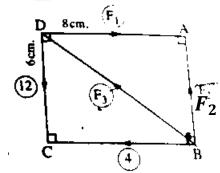
بولية مؤسس: [36] <u>In The Opposite Figure</u>: بولية مؤسسا

If the magnitude of the forces in newton and the system is in Equilibrium ,

then $F_2 = \dots$ Newton

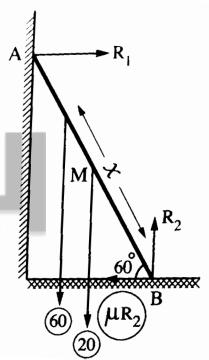


- b) 5
- (c) 3
 - d) 8

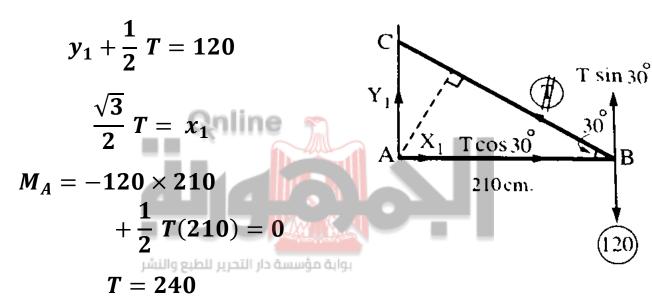


[37] A uniform ladder of weight 20 kg. wt rests at one of its ends a rough horizontal ground and with its other end against a smooth vertical wall such that the ladder equilibrium in a vertical plane , inclining to the horizontal at an angle of 60° . If the coefficient of friction between the ladder and the ground is $\frac{1}{2\sqrt{3}}$, prove that the maximum distance which a girl of weight 60~kg. wt can ascend the ladder equals half the length of the ladder

$$\mu = \frac{1}{2\sqrt{3}}$$
, $R_1 = 80$,
 $R_2 = \frac{1}{2\sqrt{3}} \times 80 = \frac{40}{\sqrt{3}}$
 $M_A = 60 \times \cos 60^\circ + 20 L \cos 60^\circ$
 $-\frac{40}{\sqrt{3}} 2L \sin 60^\circ$
 $30x - 30 L = 0$
 $x = L$
 $\left(\frac{1}{2} ladder\right)$

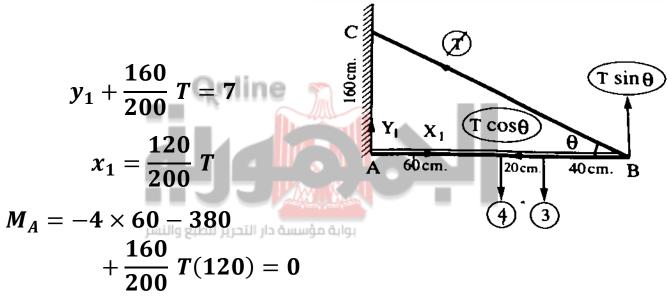


[38] \overline{AB} is a rod of negligible weight and of length 210 cm, is hanged at A to a hinge fixed at a vertical wall . It carried at B a weight of magnitude 120 newton . The rod is kept in a horizontal position by means of a light string attached at the end B of the rod , its other end is fixed at a point on the wall lying vertically above A . If the string inclined to the horizontal at an angle of measure 0° , find the magnitude of the tension in the string and the reaction of the hinge



$$y_1 = 120 - 120 = 0$$
 $x_1 = 120\sqrt{3}$
 $R = 120\sqrt{3}$

[39] A uniform rod \overline{AB} of length 120cm and of weight 4 newton , is hinged at A to a hinge fixed at a vertical wall . A weight of magnitude 3 newton is attached to the rod at a point 0cm . apart from B the rod is kept in a static equilibrium in a horizontal position by means of string attached at the end B of the rod , its other end its fixed at a point C on the wall lying vertically above A such that AC = 160cm, Find the magnitude of the tension in the string and the magnitude and the direction of the reaction of the hinge



$$T = 5$$
 $y_1 = 7 - 4 = 3$
 $x_1 = 3$
 $y_1 = 7 - 4 = 3$
 $x_1 = 3$
 $x_1 = 3$
 $x_1 = 3$

MATH ACADEMY

[40] A uniform rod rests with its upper end on a vertical wall . the coefficient of friction between the rod and the wall is equal to $\frac{1}{2}$. If the rod rests with its lower end on a horizontal plane , the coefficient of friction between the rod and the plane is equal to $\frac{3}{4}$. Find the tangent to the angle which the rod makes with horizontal when its about to slip

1

$$R_1 + \frac{1}{2} R_2 = w$$

$$\frac{3}{4} R_1 = R_2$$

$$R_1 = \frac{4}{3} R_2$$
 , $W = \frac{11}{6} R_2$

$$M_A = wL - R_2 2L \sin \theta$$

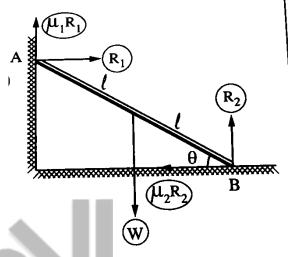
$$-\frac{1}{2} R_2 2L \cos \theta = 0$$

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$$\cos hetarac{11}{6}~R_2-2~R_2~sin~ heta-R_2cos~ heta=0$$

$$\frac{11}{6}\cos\theta-\cos\theta=2\sin\theta$$

$$\frac{5}{6}\cos\theta=2\sin\theta$$

$$tan \theta = \frac{5}{12}$$

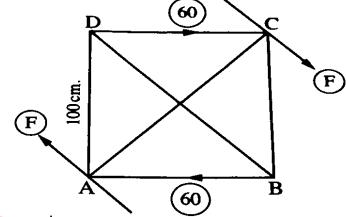


MATH ACADEMY

[41] ABCD is a square whose length is 100~cm, two forces of magnitudes 60, 60 newton act in the direction of \overrightarrow{BA} , \overrightarrow{DC} , Find two forces equal in the magnitude, acting at A and C, parallel to \overrightarrow{BD} and forming a couple equivalent to the couple formed by the first two forces

 $M_1 = M_2$ $60 \times 100 = F(100\sqrt{2})$

$$F = 30\sqrt{2}$$





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[42] If the limiting static friction force =60 newton , the resultant reaction force equals 100 newton , then the normal reaction force =......newton

- a) 60
- (b) 80
 - c) 100
 - d) 200

MATH ACADEMY

[43] A body of weight 21 newton is placed on a rough horizontal plane , two horizontal forces act on the body of magnitudes 3 newton and 5 newton and include an angle of measure 60° , if the body is about to move , then the coefficient of static friction equal

- a) $\frac{3}{7}$
- b) $\frac{1}{7}$
- $\left(c\right)\frac{1}{3}$
 - d) $\frac{3}{5}$

Galine

[44] A body of weight (w) is placed on rough plane inclined to the horizontal at an angle of measure (θ) . If the least force acting parallel to the line of the greatest slope of the plane to make the body about to move upwards the plane equals $(2 \ w \ sin \ \theta)$, Prove that:

- a) the measure of the angle of friction = θ
- b) The magnitude of the resultant reaction force = w

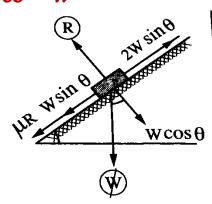
$$2w \sin \theta = w \sin \theta + \mu w \cos \theta$$

$$\sin \theta = \mu \cos \theta , \tan \theta = \tan \lambda , \theta = \lambda$$

$$R' = R\sqrt{1 + \tan^2 \lambda} = R' =$$

$$w \cos \theta \sqrt{1 + \tan^2 \theta}$$

$$w \cos \theta - \sec \theta = W$$



[45] If the force $\vec{F}=2\hat{\imath}-\hat{\jmath}+3\hat{k}$ acts at the point (-3,1,2), Find the moment vector of the force \vec{F} about the point B(2,2,-1), then calculate the length of the perpendicular drawn from the point B on the line of action of the force

$$M = \begin{vmatrix} i & j & k \\ -5 & -1 & 3 \\ 2 & -1 & 3 \end{vmatrix} = o\vec{i} + 21\vec{j} + 7\vec{k}$$
$$L = \frac{\|M\|}{\|F\|} = \frac{\sqrt{21^2 + 7^2}}{\sqrt{4 + 1 + 9}}$$

[46] The forces $\overrightarrow{F_1} = l\vec{\imath} + m\vec{\jmath}$, $\overrightarrow{F_2} = \vec{\imath} - 3\vec{\jmath}$, $\overrightarrow{F_3} = -2\vec{\imath} + \vec{\jmath}$ act at the point A(1,2), B(0,4), C(2,4) respectively. If the sum of the moments of the forces about the origin point = -9 and the sum of the moments of the forces about the point D(-2,3) equals $-4\vec{k}$ Find the value of each of l and m

$$(3,-1) \times (l,m) + (2,1) \times (1,-3) + (4,1) \times (-2,1) = -4$$

$$3m+L-6-1+4+2=-4$$
, $3m+L=-3$

At Original Point = -9

$$(\mathbf{1}$$
 , $\mathbf{2}$) $imes$ $(\mathbf{l}$, \mathbf{m}) $+$ $(\mathbf{0}$, $\mathbf{4}$) $imes$ $(\mathbf{1}$, $-\mathbf{3}$) $+$ $(\mathbf{2}$, $\mathbf{4}$) $imes$ $(-\mathbf{2}$, $\mathbf{1}$)

$$m-2l-4+2+8=-9$$

$$m-2l=-15$$
 , $L=6$, $M=-3$

[47] If a set forces are in equilibrium, then

- a) only the sum of the moments of the forces about any point vanishes
- b) Only the resultant of the forces vanish
- c) The sum of the moments of the forces about any point vanishes and the resultant of the forces vanish
 - d) The resultant of the forces equals the sum of the magnitudes of the forces and the sum of the moments of the forces about any point are not vanishes

[48] A body of weight 36 newton is placed on a rough horizontal plane. If the coefficient of the static friction between the body and the plane equals $\frac{1}{3}$ and horizontal force acts on the body trying to move it, then the magnitude of the friction force \in

a)
$$]\frac{1}{3}$$
, 12]

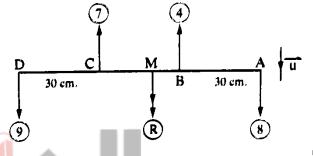
b)
$$]\frac{1}{3}$$
, 36]

[49] A, B, C and d are four different points lying on a straight line where : AB = BC = CD = 30cm two forces of magnitudes 8, 9 Newton act at the points A and D respectively and in the same direction perpendicular to the straight line Another two forces of magnitudes 4, 7 newton act at the points B and C respectively in the opposite direction of the first two force. Find the resultant of these forces and the distance between the point of action of the resultant

and A

$$R = 6 N$$
,
 $-4 \times 30 - 7 \times 60 + 9 \times 90$
 $= 6 \times AM$

AM = 45cm, $M \in \overline{AD}$, MA = 45cm



[50] A uniform rod of length 4 meters and weight 50 kg. wt rests horizontally on two supports at its ends. if a weight of magnitude 20 kg. wt is fixed at 1 meter a part from one of its ends, Find the reaction of the two supports

The rod is in equilibrium

$$R_1+R_2=70$$
 , $M_A=0$
$$20\times 1+50\times 2-R_2\times 4=0$$

$$R_2=30~kg~.wt~, R_1=40~kg.wt$$

